CHAPTER 1.1 AND 1.2

Making Conjectures: Inductive Reasoning and

Exploring the Validity of Conjectures

BY THE END OF THIS LESSON YOU WILL BE ABLE TO…

- Make conjectures by observing patterns and identifying properties, and justify the reasoning.
- Provide examples of how inductive reasoning might lead to false conclusions.
- Critique the following statement "Decisions can be made and actions taken based upon inductive reasoning".

WHAT DOES THAT MEAN?

- You will learn about the following:
- “Conjecture”
- “Inductive Reasoning”
- How to make a conjecture
- How to use inductive reasoning to prove a statement
- How to determine whether a conjecture is correct or incorrect

HOW MANY PEOPLE HERE HAVE TAKEN SCIENCE?

- Have you heard of a hypothesis?
- A hypothesis is a proposed explanation that is made using limited information and is a starting point for further investigation.
- A conjecture is very similar:
- A conjecture is an expression or statement that is based on available information, but is not yet proved.
- A conjecture is essentially a mathematical hypothesis.

CONJECTURES

- A quick way to make a conjecture is: “I believe ____________ to be true, but I don’t have enough evidence yet.”
- Try making a conjecture about this:

- How many stars do you think would be in diagram #4?

CONJECTURE OF THE STARS

- Let's write out our information first:
- In the first diagram there is one star.
- In the second diagram there are four stars.
- In the third figure there are sixteen stars.
- Any thoughts now?
- 16? Why do you say that?

INDUCTIVE REASONING

- There will be sixteen in the next diagram because the number of stars in each diagram increase by a scale factor of 4 each time.
- Guess what, you just used inductive reasoning to come up with a conjecture.
- Inductive reasoning is coming to a conclusion by using patterns and noticing properties in specific examples.
- "As a result of ___________ I believe ____________."

INDUCTIVE REASONING

- Can you think of any examples of where you have either used or have seen inductive reasoning used?

- Imagine you’re a detective and you’re trying to find a serial killer. So far there have been three victims. Each one was female, had blonde hair, and was in her mid-twenties. Use inductive reasoning to come up with a conjecture about the killer’s next victim.

IS IT TRUE?

- Is a conjecture true if you use inductive reasoning to support your conjecture?

- Not always. Just because you have evidence to support your theory doesn’t mean that it is necessarily true. The larger the amount of evidence you have, the stronger your conjecture will be, but it still doesn’t prove it.
- There will be times when your conjecture will be proven to be wrong after more evidence is gathered.

EXAMPLES OF CONJECTURES

Make a conjecture about the product of two odd integers.

Jay's Solution

$(+3)(+7) = (+21)$ — Odd integers can be negative or positive. I tried two positive odd integers first. The product was positive and odd.

$(-5)(-3) = (+15)$ — Next, I tried two negative odd integers. The product was again positive and odd.

$(+3)(-3) = (-9)$ — Then I tried the other possible combination: one positive odd integer and one negative odd integer. This product was negative and odd.

My conjecture is that the product of two odd integers is an odd integer. — I noticed that each pair of integers I tried resulted in an odd product.

$(-211)(-17) = (+3587)$ — I tried other integers to test my conjecture. The product was again odd.

EXAMPLE 3 **Using inductive reasoning to develop a conjecture about perfect squares**

Make a conjecture about the difference between consecutive perfect squares.

Steffan's Solution: Comparing the squares geometrically

I represented the difference using unit tiles for each perfect square. First, I made a 3 × 3 square in orange and placed a yellow 2 × 2 square on top. When I subtracted the 2 × 2 square, I had 5 orange unit tiles left.

Next, I made 3 × 3 and 4 × 4 squares. When I subtracted the 3 × 3 square, I was left with 7 orange unit tiles. I decided to try greater squares.

My conjecture is that the difference between consecutive squares is always an odd number.

I saw the same pattern in all my examples: an even number of orange unit tiles bordering the yellow square, with one orange unit tile in the top right corner. So, there would always be an odd number of orange unit tiles left, since an even number plus one is always an odd number.

I tested my conjecture with the perfect squares 7 × 7 and 8 × 8. The difference was an odd number.

The example supports my conjecture.

Francesca's Solution: Describing the difference numerically

$2^2 - 1^2 = 4 - 1$
$2^2 - 1^2 = 3$

I started with the smallest possible perfect square and the next greater perfect square: 1^2 and 2^2. The difference was 3.

$4^2 - 3^2 = 7$
$9^2 - 8^2 = 17$

Then I used the perfect squares 3^2 and 4^2. The difference was 7. So, I decided to try even greater squares.

My conjecture is that the difference between consecutive perfect squares is always a prime number.

I thought about what all three differences—3, 7, and 17—had in common. They were all prime numbers.

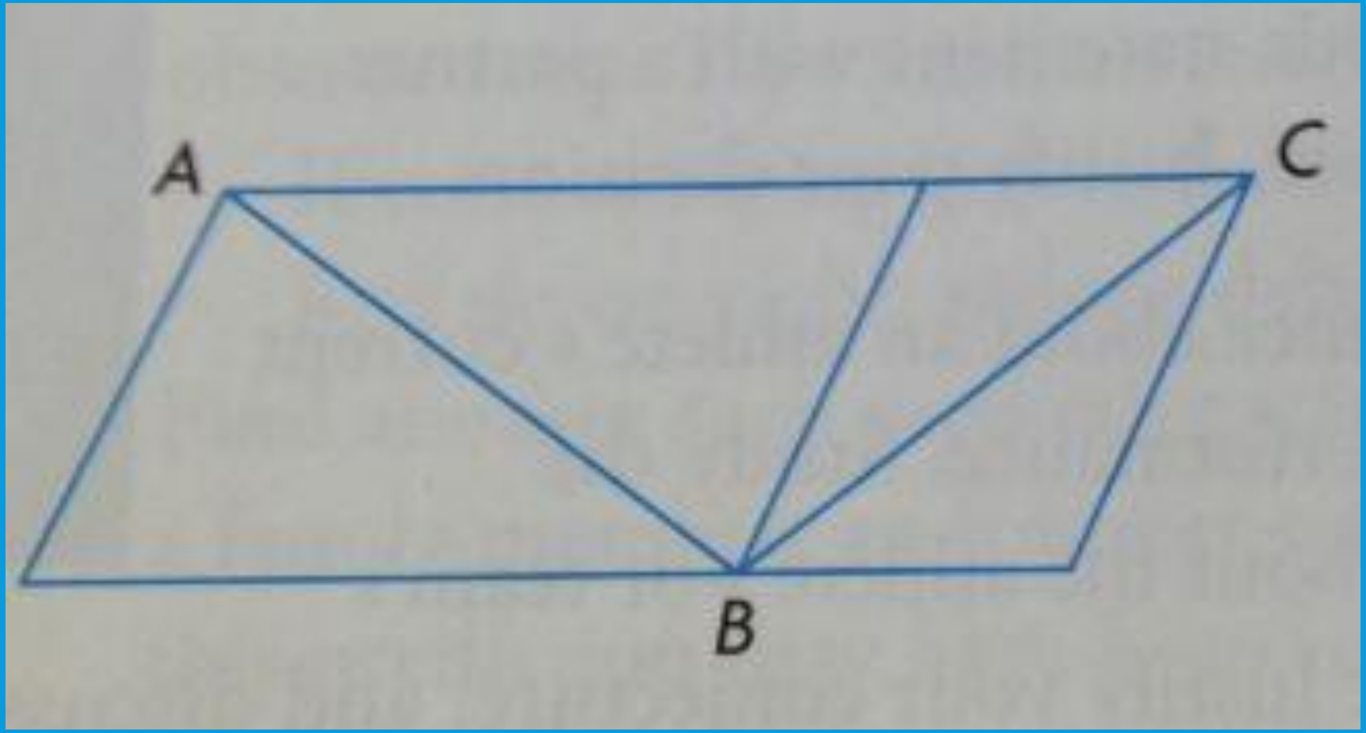
$12^2 - 11^2 = 23$

To test my conjecture, I tried the perfect squares 11^2 and 12^2. The difference was a prime number.

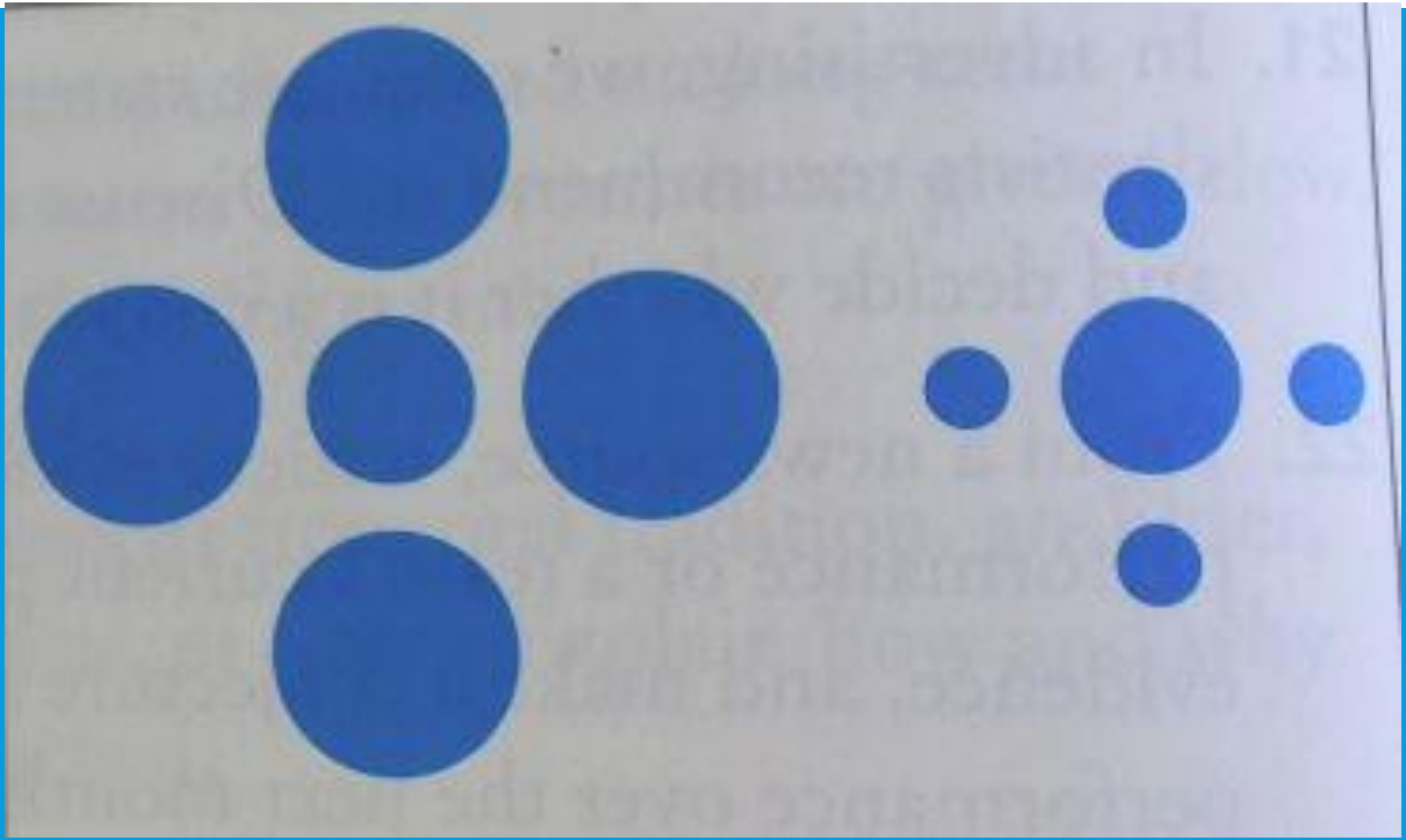
The example supports my conjecture.

MORE EXAMPLES

Come up with a conjecture about the diagonal lines AB and BC.



MAKE A CONJECTURE ABOUT THE MIDDLE CIRCLES



ASSIGNMENT

- Page 12-14, Questions 1, 2, 3, 6, 8, 11, 13, and 16
- Page 17, Questions 1-3