

Using Reasoning to Find a Counterexample to a Conjecture

LAST CLASS...

- We looked at making conjectures and using inductive reasoning to support them.
- We saw that not all conjectures with evidence found through inductive reasoning were true.
- Today we will look at a way to prove a conjecture is not true.

AFTER TODAY'S LESSON YOU WILL BE ABLE TO...

- Analyze an inductive argument for its validity.
- This means that you will be able to determine whether an argument is true or false through the use of counterexamples.

COUNTEREXAMPLES

- What is a counterexample?
- A counterexample is an example that proves a conjecture cannot be true.
- What does this look like?

COUNTEREXAMPLE EXAMPLE

- Bob makes the conjecture: The sum of any two composite numbers (numbers that have more than one divisor) will also be a composite number.
- Bob's evidence:
- 2 + 2 = 4
- 2 + 4 = 6
- 4 + 6 = 10.
- Do you agree with Bob?

COUNTEREXAMPLE

- Wait, how about 8 + 9?
- That gives us 17.
- 17 is a prime number, not composite.
- This means we have come up with an example that proves Bob's conjecture was incorrect.
- "8 + 9 = 17" is a counterexample.

ANOTHER EXAMPLE

- Last class we looked at an example that involved a girl named Francesca. She made the conjecture: The difference between two consecutive squares will be a prime number.
- She used 1 and 2, 3 and 4, and 8 and 9.
- Let's ignore her work and try this ourselves.
- $1^2 = 1$ $2^2 = 4$ 4 1 = 3
- $\cdot 2^2 = 4$ $3^2 = 9$ 9 4 = 5
- $3^2 = 9$ $4^2 = 16$ 16 9 = 7
- $4^2 = 16$ $5^2 = 25$ 25 16 = 9

FRANCESCA'S CONJECTURE

$\cdot 4^2 = 16$ $5^2 = 25$ 25 - 16 = 9

- 9 is not a prime number. We can now use this as our counterexample to prove that her conjecture is not true.
- In her evidence, Francesca skipped over consecutive squares and as a result missed the example that would prove her conjecture not to be true.
- Remember, we may not always find a counterexample right away, but it doesn't mean our conjecture is necessarily true.

WHAT IS THE POINT OF COUNTEREXAMPLES?

- Counterexamples help us to disprove conjectures, but they aren't made to make us feel bad or stupid.
- Yes, they let us know what we did wrong, but they also help us by showing us we just need to revise our conjecture.
- It could mean that we just forget the whole conjecture or just edit the phrasing of our conjecture.
- It can help lead to a more detailed conjecture.

LET'S TRY ONE MORE EXAMPLE

- Question 1 on page 22 of the text book asks you to show that each statement is false by finding a counterexample. Let's try some of these together...
- A) A number that is not negative is positive.
- B) All prime numbers are odd.
- D) The height of a triangle lies inside the triangle.

YOURTURN

- Pages 22-24
- Questions 2, 3, 4, 6, 7, 8, 9, 13, 14, 17