

5.4

Graphing Linear Inequalities in Two Variables

5.4 OBJECTIVES

1. Graph linear inequalities in two variables
2. Graph a region defined by linear inequalities

What does the solution set look like when we are faced with an inequality in two variables? We will see that it is a set of ordered pairs best represented by a shaded region. Recall that the general form for a linear inequality in two variables is

$$ax + by < c$$

in which a and b cannot both be 0. The symbol $<$ can be replaced with $>$, \leq , or \geq . Some examples are

$$y < -2x + 6 \quad x - 2y > 4 \quad \text{or} \quad 2x - 3y \geq x + 5y$$

As was the case with an equation, the solution set of a linear inequality in two variables is, a set of ordered pairs of real numbers. However, in the case of the linear inequalities, we will find that the solution sets will be all the points in an entire region of the plane, called a **half plane**.

To determine such a solution set, let's start with the first inequality listed above. To graph the solution set of

$$y < -2x + 6$$

we begin by writing the corresponding linear equation

$$y = -2x + 6$$

First, note that the graph of $y = -2x + 6$ is simply a straight line.

Now, to graph the solution set of $y < -2x + 6$, we must include all ordered pairs that satisfy that inequality. For instance, if $x = 1$, we have

$$y < -2 \cdot 1 + 6$$

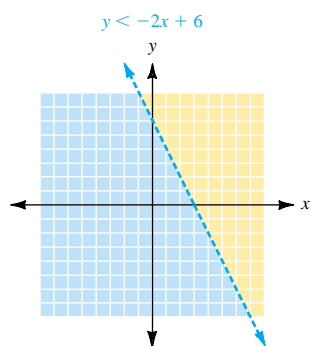
$$y < 4$$

So we want to include all points of the form $(1, y)$, in which $y < 4$. Of course, because $(1, 4)$ is *on* the corresponding line, this means that we want all points *below* the line along the vertical line $x = 1$. The result will be similar for any choice of x , and our solution set will then contain all points below the line $y = -2x + 6$. We can then graph the solution set as the shaded region shown. We have the following definition.



CAUTION

Note that we have **two** variables. This is a different process than we discussed in Section 4.4, when we had only **one** variable.



NOTE The line is dashed to indicate that the points do not represent solutions.

NOTE We call the graph of the equation $ax + by = c$ the **boundary line** of the half plane.

Definitions: Solution Set of an Inequality

In general, the solution set of an inequality of the form

$$ax + by < c \quad \text{or} \quad ax + by > c$$

will be a half plane either above or below the corresponding line determined by

$$ax + by = c$$

How do we decide which half plane represents the desired solution set? The use of a **test point** provides an easy answer. Choose any point *not* on the line. Then substitute the coordinates of that point into the given inequality. If the coordinates satisfy the inequality (result in a true statement), then shade the region or half plane that includes the test point; if not, shade the opposite half plane. Example 1 illustrates the process.

Example 1

Graphing a Linear Inequality

Graph the linear inequality

$$x - 2y < 4$$

First, we graph the corresponding equation

$$x - 2y = 4$$

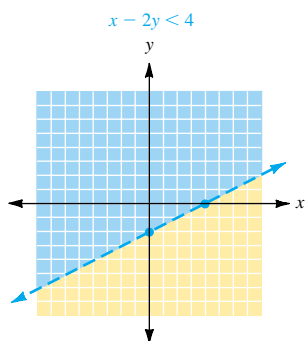
to find the boundary line. Now to decide on the appropriate half plane, we need a test point *not* on the line. As long as the line *does not pass through the origin*, we can always use $(0, 0)$ as a test point. It provides the easiest computation.

Here letting $x = 0$ and $y = 0$, we have

$$0 - 2 \cdot 0 \stackrel{?}{<} 4$$

$$0 < 4$$

Because this is a true statement, we proceed to shade the half plane including the origin (the test point), as shown.



NOTE The boundary line is dashed to indicate that any point **on** the line itself is **not** a solution.



CHECK YOURSELF 1

Graph the solution set of $3x + 4y > 12$.

The graphs of some linear inequalities will include the boundary line. That will be the case whenever equality is included with the inequality statement, as illustrated in Example 2.

Example 2

Graphing a Linear Inequality

Graph the solution set of

$$y \leq 2x$$

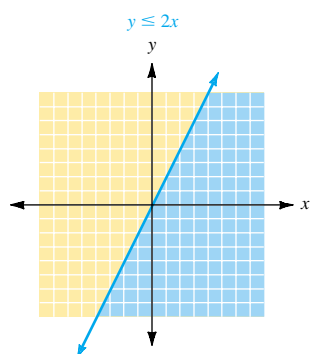
We proceed as before by graphing the boundary line (it is solid because equality is included). The only difference between this and previous examples is that we *cannot use the origin* as a test point. Do you see why?

Choosing $(1, 1)$ as our test point gives the statement

$$1 \stackrel{?}{\leq} 2 \cdot 1$$

$$1 \leq 2$$

Because the statement is *true*, we shade the half plane *including* the test point $(1, 1)$.



NOTE The choice of $(1, 1)$ is arbitrary. We simply want any point *not* on the line.

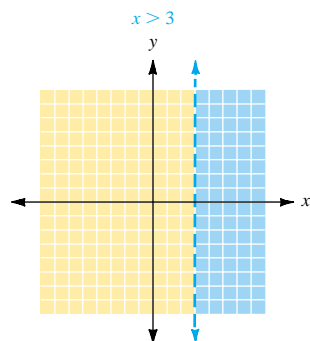


CHECK YOURSELF 2

Graph the solution set of $3x + y > 0$.

Let's consider a special case of graphing linear inequalities in the rectangular coordinate system.

NOTE Here we specify the rectangular coordinate system to indicate we want a two-dimensional graph.



Example 3

Graphing a Linear Inequality

Graph the solution set of $x > 3$.

First, we draw the boundary line (a dashed line because equality is not included) corresponding to

$$x = 3$$

We can choose the origin as a test point in this case, and that results in the false statement

$$0 > 3$$

We then shade the half plane *not* including the origin. In this case, the solution set is represented by the half plane to the right of the vertical boundary line.

As you may have observed, in this special case choosing a test point is not really necessary. Because we want values of x that are *greater than* 3, we want those ordered pairs that are to the *right* of the boundary line.



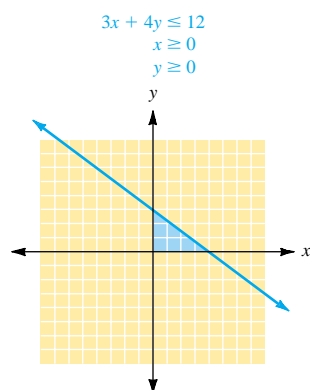
CHECK YOURSELF 3

Graph the solution set of

$$y \leq 2$$

in the rectangular coordinate system.

Applications of linear inequalities will often involve more than one inequality condition. Consider Example 4.



Example 4

Graphing a Region Defined by Linear Inequalities

Graph the region satisfying the following conditions.

$$3x + 4y \leq 12$$

$$x \geq 0$$

$$y \geq 0$$

The solution set in this case must satisfy *all three conditions*. As before, the solution set of the first inequality is graphed as the half plane *below* the boundary line. The second and third inequalities mean that x and y must also be nonnegative. Therefore, our solution set is restricted to the first quadrant (and the appropriate segments of the x and y axes), as shown.



CHECK YOURSELF 4

Graph the region satisfying the following conditions.

$$3x + 4y < 12$$

$$x \geq 0$$

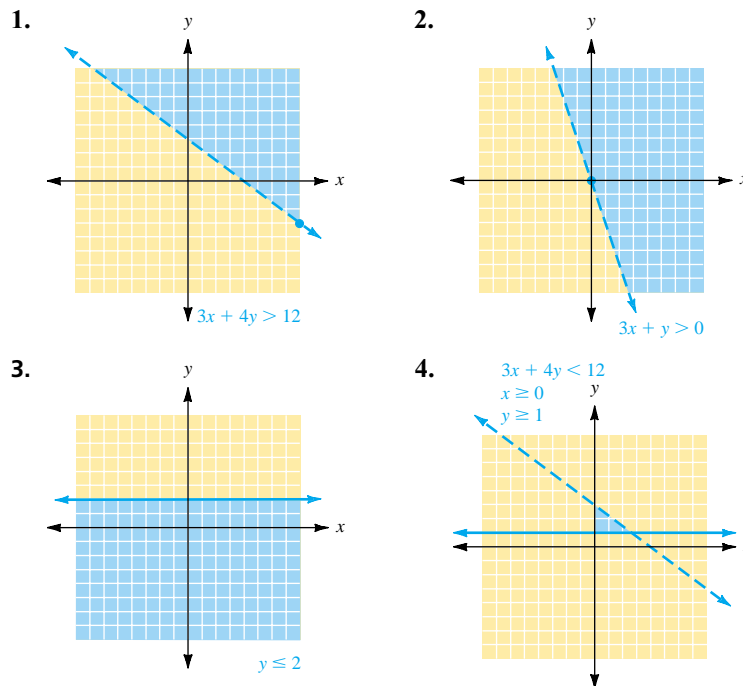
$$y \geq 1$$

The following algorithm summarizes our work in graphing linear inequalities in two variables.

Step by Step: Graphing a Linear Inequality

- Step 1** Replace the inequality symbol with an equality symbol to form the equation of the boundary line of the solution set.
- Step 2** Graph the boundary line. Use a dashed line if equality is not included ($<$ or $>$). Use a solid line if equality is included (\leq or \geq).
- Step 3** Choose any convenient test point *not* on the boundary line.
- Step 4** If the inequality is *true* for the test point, shade the half plane *including* the test point. If the inequality is *false* for the test point, shade the half plane *not including* the test point.

CHECK YOURSELF ANSWERS

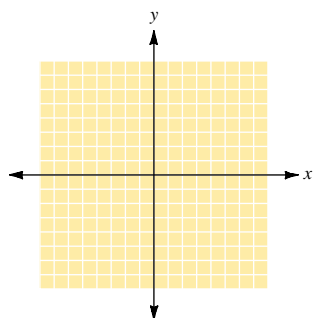




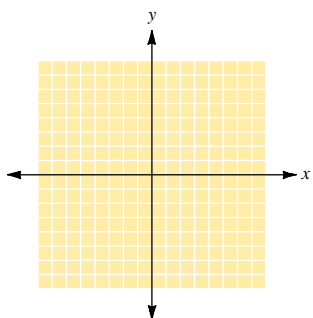
Exercises

In exercises 1 to 24, graph the solution sets of the linear inequalities.

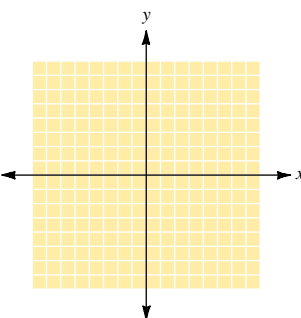
1. $x + y < 4$



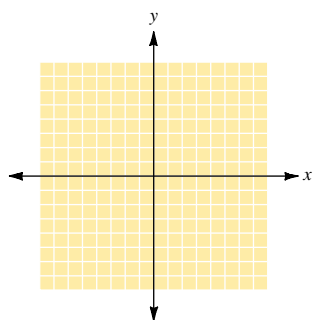
2. $x + y \geq 6$



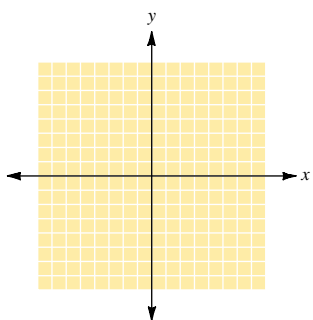
3. $x - y \geq 3$



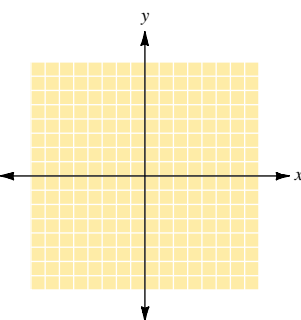
4. $x - y < 5$



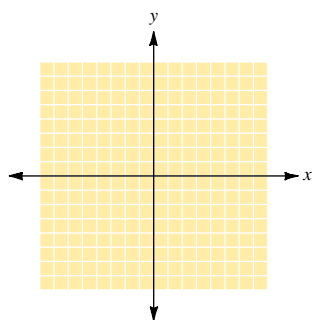
5. $y \geq 2x + 1$



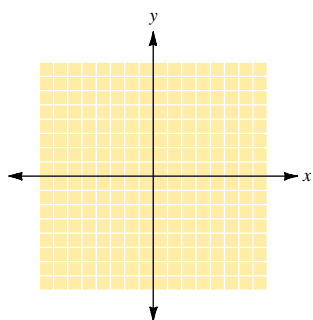
6. $y < 3x - 4$



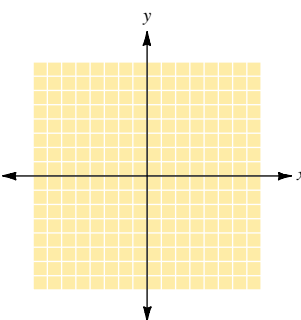
7. $2x + 3y < 6$



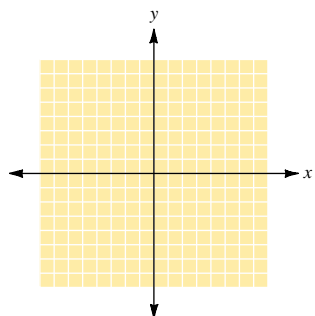
8. $3x - 4y \geq 12$



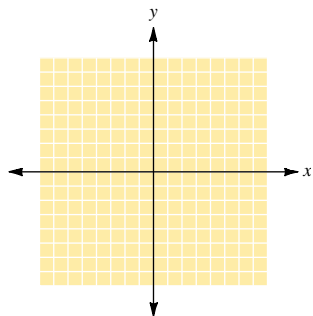
9. $x - 4y > 8$



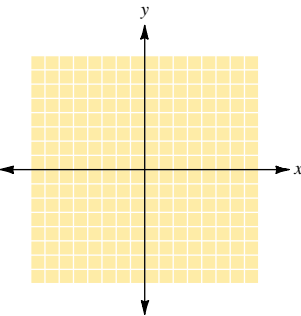
10. $2x + 5y \leq 10$



11. $y \geq 3x$



12. $y \leq -2x$



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ANSWERS

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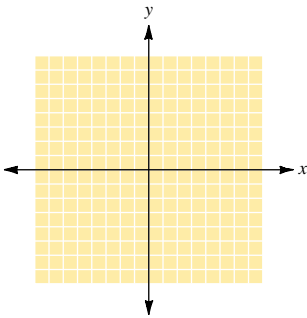
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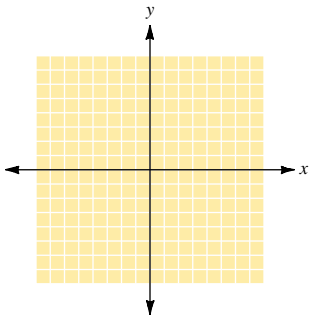
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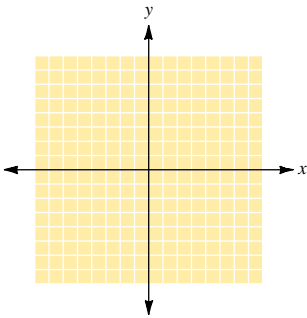
13. $x - 2y > 0$



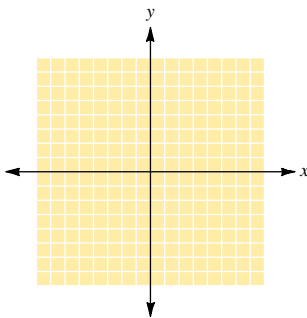
14. $x + 4y \leq 0$



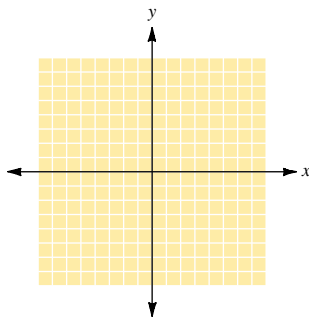
15. $x < 3$



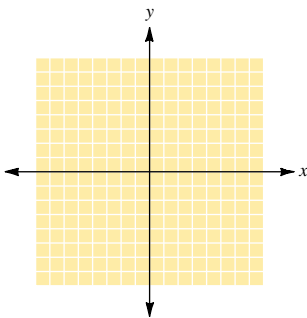
16. $y < -2$



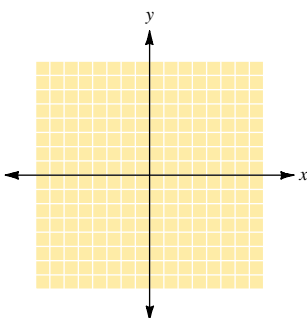
17. $y > 3$



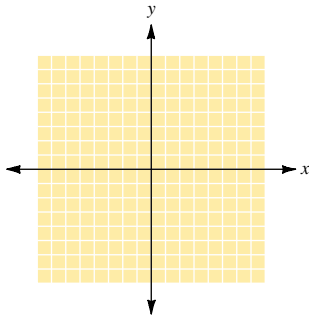
18. $x \leq -4$



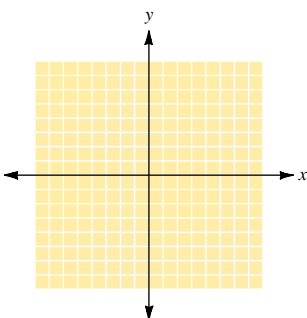
19. $3x - 6 \leq 0$



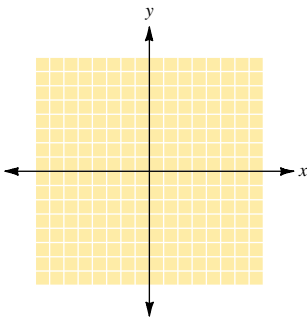
20. $-2y > 6$



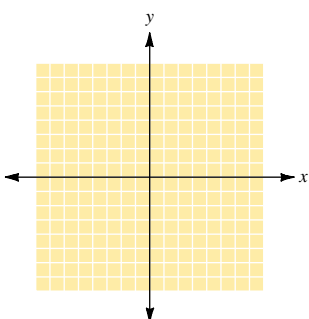
21. $0 < x < 1$



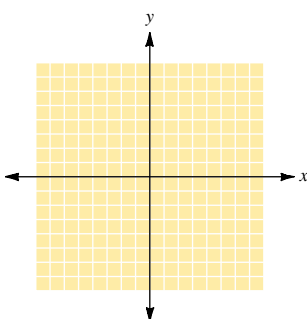
22. $-2 \leq y \leq 1$



23. $1 \leq x \leq 3$

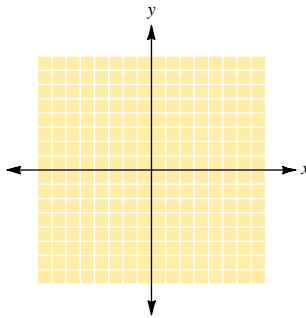


24. $1 < y < 5$

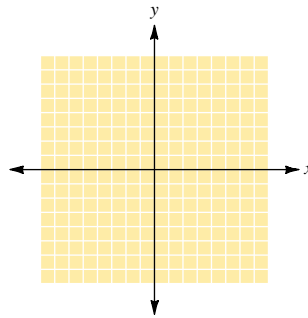


In exercises 25 to 28, graph the region satisfying each set of conditions.

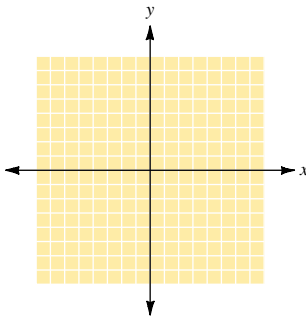
25. $0 \leq x \leq 3$
 $2 \leq y \leq 4$



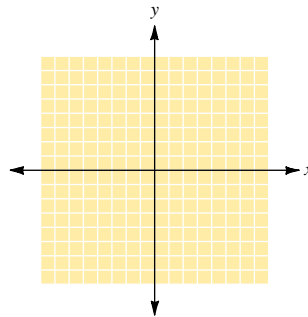
26. $1 \leq x \leq 5$
 $0 \leq y \leq 3$



27. $x + 2y \leq 4$
 $x \geq 0$
 $y \geq 0$



28. $2x + 3y \leq 6$
 $x \geq 0$
 $y \geq 0$



29. Assume that you are working only with the variable x . Describe the set of solutions for the statement $x > -1$.



30. Now, assume that you are working in two variables, x and y . Describe the set of solutions for the statement $x > -1$.



31. **Manufacturing.** A manufacturer produces a standard model and a deluxe model of a 13-in. television set. The standard model requires 12 h to produce, and the deluxe model requires 18 h. The labor available is limited to 360 h per week.

If x represents the number of standard-model sets produced per week and y represents the number of deluxe models, draw a graph of the region representing the feasible values for x and y . (This region will be the solution set for the system of inequalities.) Keep in mind that the values for x and y must be nonnegative because they represent a quantity of items.

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26. _____

27. _____

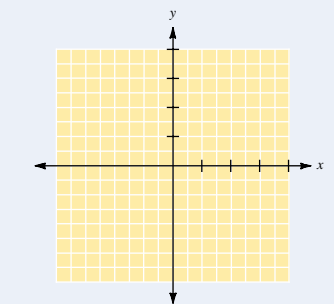
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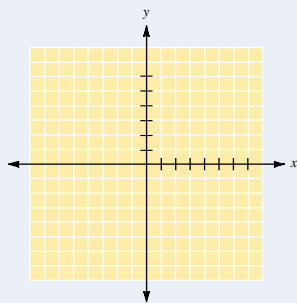
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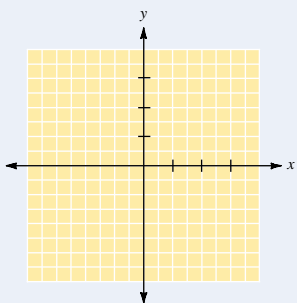
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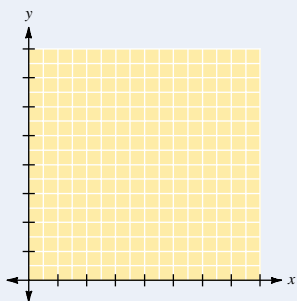
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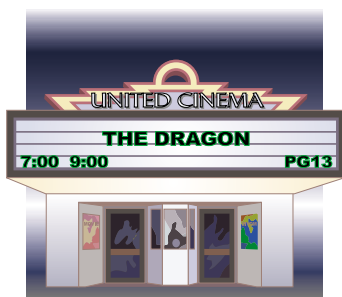
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- 32. Manufacturing.** A manufacturer produces portable radios and CD players. The radios require 10 h of labor to produce and CD players require 20 h. Let x represent the number of radios produced and y the number of CD players.

If the labor hours available are limited to 300 h per week, graph the region representing the feasible values for x and y .

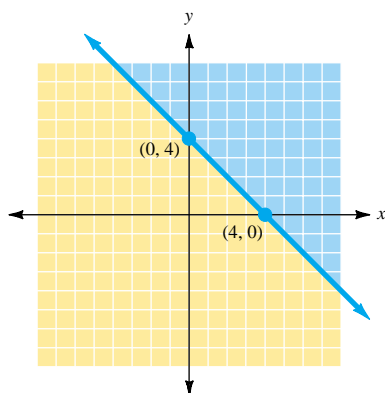
- 33. Serving capacity.** A hospital food service can serve at most 1000 meals per day. Patients on a normal diet receive 3 meals per day and patients on a special diet receive 4 meals per day. Write a linear inequality that describes the number of patients that can be served per day and draw its graph.

- 34. Time on job.** The movie and TV critic for the local radio station spends 3 to 7 hours daily reviewing movies and fewer than 4 hours reviewing TV shows. Let x represent the hours watching movies and y represent the time spent watching TV. Write two inequalities that model the situation, and graph their intersection.

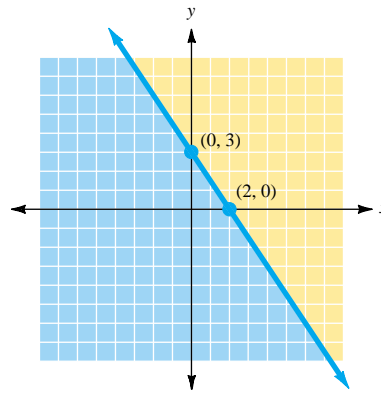


In exercises 35 to 38, write an inequality for the shaded region shown in the figure.

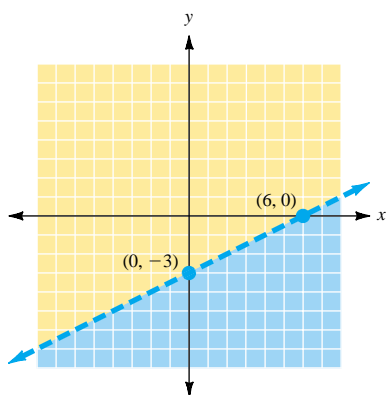
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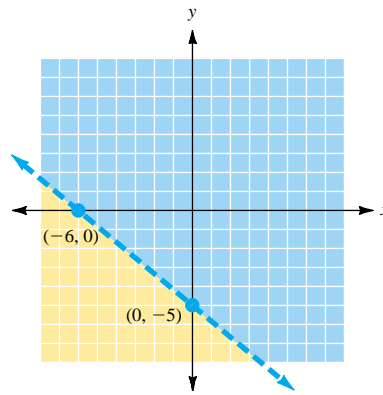
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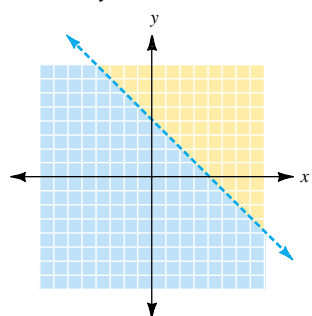


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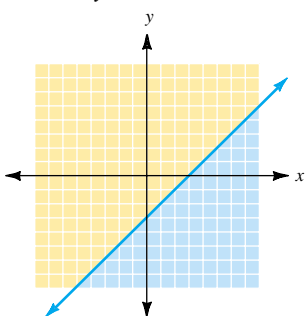
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Answers

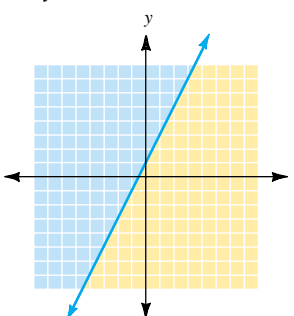
1. $x + y < 4$



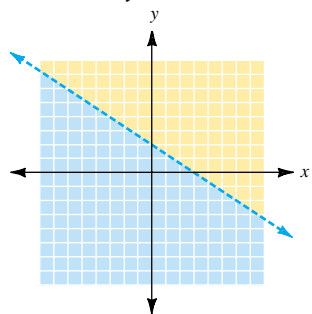
3. $x - y \geq 3$



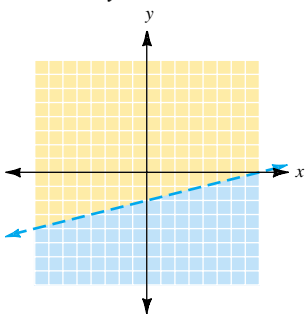
5. $y \geq 2x + 1$



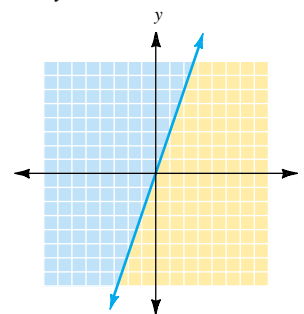
7. $2x + 3y < 6$



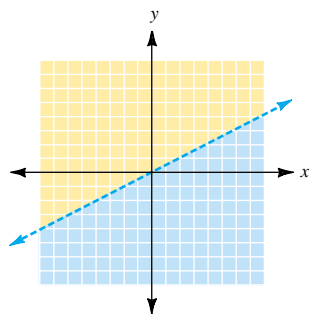
9. $x - 4y > 8$



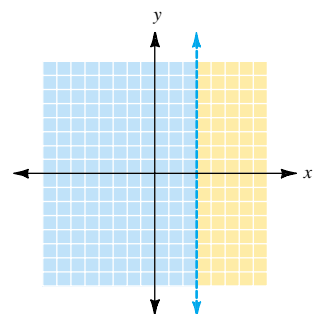
11. $y \geq 3x$



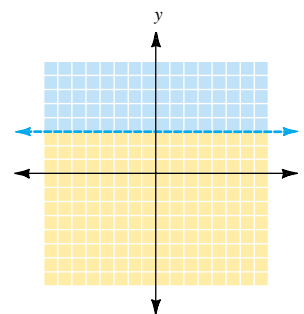
13. $x - 2y > 0$



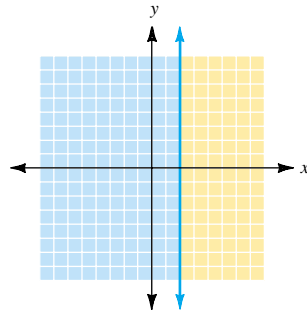
15. $x < 3$



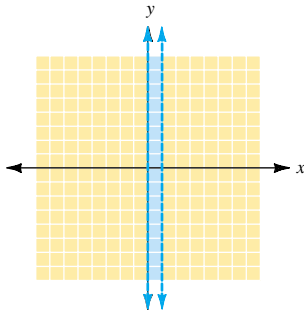
17. $y > 3$



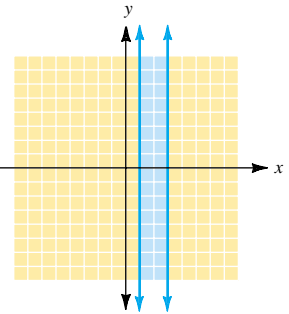
19. $3x - 6 \leq 0$



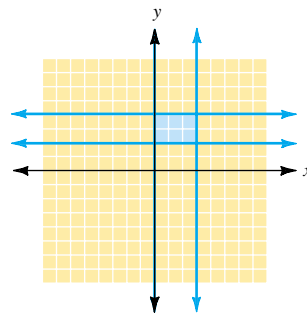
21. $0 < x < 1$



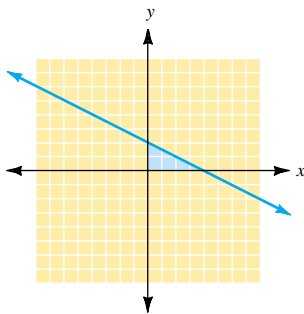
23. $1 \leq x \leq 3$




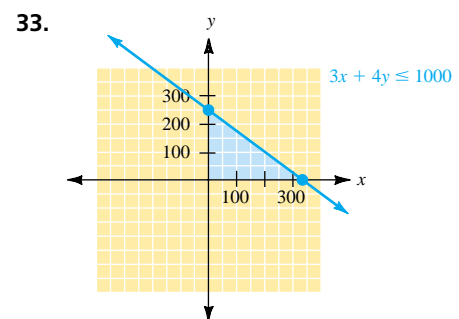
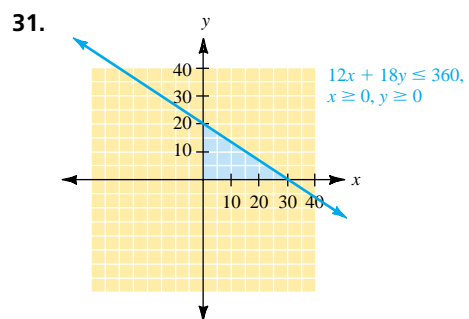
25. $0 \leq x \leq 3$
 $2 \leq y \leq 4$



27. $x + 2y \leq 4$
 $x \geq 0$
 $y \geq 0$



29. 



35. $y \geq -x + 4$

37. $y < \frac{1}{2}x - 3$