# 6.5 - Optimization Problems II

Wednesday, April 29, 2015 9:06 AM

**6.5- Optimization Problems II- Exploring Solutions**

Keep in Mind:

- The value of the objective function for a system of linear inequalities varies throughout the feasible region, but in a predictable way.
- The boundary intersection points are the vertices (corners) of the feasible region.
- These vertices (corners) represent the optimal solutions for the objective function
- If a vertex is not part of the solution set, then the optimal solution may be found nearby
- To verify an optimal solution, substitute its coordinates into each inequality in the system and evaluate.

Constraint: boundaries, inequalities

Feasible Region: Solution set, "double-shaded" area

Objective Function: equation, what we actually want to know, want a max or min from it

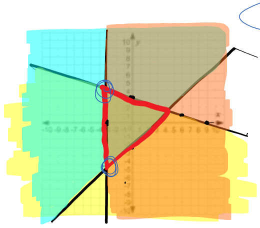
Maximize/Minimize: very most, very least, found "on the corners"

**Example.** Restrictions: $x, y \in R$
Constraints: $x + 3y \le 9$, $x - y \le 3$, $x \ge -3$
Objective function: $P = 2x + y$

$x + 3y \le 9$, $3y \le -x + 9$, $y \le -\frac{1}{3}x + 3$; $x - y \le 3$, $-y \le -x + 3$, $y \ge x - 3$

a) Draw a graph to model the situation
b) What point in the feasible region would result in the maximum value of the objective functions?
c) What point in the feasible region would result in the minimum value of the objective function?

$y \le -\frac{1}{3}x + 3$, $y \ge x - 3$, $x \ge -3$



$x + 3y \le 9$, $0 + 3(0) \le 9$, $0 \le 9$ ✓   $x - y \le 3$, $0 - 0 \le 3$, $0 \le 3$ ✓   $0 \ge -3$ ✓

$-\frac{1}{3}x + 3 = x - 3$

$6 = \frac{4}{3}x$

$4.5 = x$

$4.5 - y \le 3$, $-y \le -1.5$, $y \ge 1.5$ → $(4.5, 1.5)$

$x - y \le 3$, $-3 - y \le 3$, $-y \le 6$, $y \ge -6$ → $(-3, -6)$

$x + 3y \le 9$, $-3 + 3y \le 9$, $3y \le 12$, $y \le 4$ → $(-3, 4)$

If Intersection Points aren't clear:

1. Slope-intercept form
2. Set two lines equal to each other
3. Solve for x
4. Sub x-value back into each equation to find y.

$P = 2x + y$
$P = 2(4.5) + 1.5$
$P = 9 + 1.5$
$P = 10.5$ ← max

$P = 2x + y$
$P = 2(-3) + 4$
$P = -6 + 4$
$P = -2$

$P = 2x + y$
$P = 2(-3) + -6$
$P = -6 + -6$
$P = -12$ ← min

**Example.** Kathy and Ravi volunteer at the local food bank each weekend. Kathy can work no more than 10 hours per weekend. Ravi can work no more than 12 hours per weekend. The food bank can assign both of them for 18 hours or less per weekend. Kathy can fill 10 boxes with food in 1 hour, while Ravi can fill 12 boxes per hour. The food bank wants to maximize the number of boxes filled in one weekend by these two volunteers.

What are the variables? Represent them with a letter

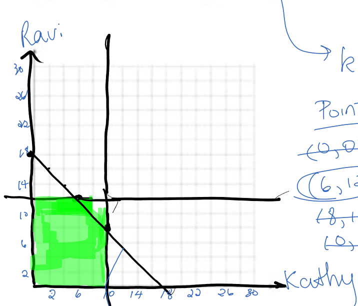
Let K = Kathy hour, R = Ravioli hour, F = food boxes

Write out inequalities using these variables.

$K \le 10$, $k \ge 0$   $R \le 12$, $r \ge 0$   $k + r \le 18$ → $k = -r + 18$

Optimize: $10k + 12r = F$

Graph and determine the equations.



Points

~~(0, 0)~~   ~~$10(0) + 12(0) = F$, $0 = F$~~

(6, 12)   $10(6) + 12(12) = F$, $60 + 144 = F$, $204 = F$

~~(8, 10)~~   $10(8) + 12(10) = F$, $80 + 120 = F$, ~~$200 = F$~~

~~(0, 12)~~   $10(0) + (12)(12) = F$, $0 + \cancel{144} = F$, ~~$144 = F$~~

Max food is at 6 hrs for Kathy and 12 hours for Ravi.

**Example.** A BC farmer wants to plant a combination of apple and pear trees that will maximize revenue.

She wants to plant no more than 500 trees altogether.
She wants to plant at least four times as many apple as pear trees
The yield per apple tree is 4 bushels, and the yield per pear tree is 3 bushels.
Apples pay the farmer $8.75 per bushel and pears pay 9.50 per bushel.