

UNIT 1: PROPORTIONAL REASONING

Chapter 8

8.0 - REVIEW

Rate

- A measurement expressed in terms of another measurement
 - Km/h
 - Litres per second

Equation

- A math expression with an equals sign in it

8.0 - REVIEW

Linear Function

- Two variables related to each other in an equation

- $f(x) = 3x - 5$

- $d(t) = -6t + 8$

▪ Square Root

- What holds up square trees

8.0 - REVIEW

Square Root (for real)

- The “Vans” sign $\sqrt{x} = (x)^{1/2}$

Cube Root

- The “3” is in the notch of the root sign $\sqrt[3]{x} = (x)^{1/3}$

8.0 - REVIEW

Reciprocals

- Two fractions that are the same, except flipped
 - $2/3$ and $3/2$
 - $1/2$ and $\frac{2}{1}$

Expression

- Math stuff with NO equals sign $3x + 5$ ~~not an equation~~

8.0 - REVIEW

Ratio

- Two values that are related to each other, expressed as $\#:\#$
- 3:2

8.0 - REVIEW

$$\overset{1}{(3, 2)}$$

$$\overset{2}{(5, 4)}$$

$$m = \frac{4 - 2}{5 - 3}$$

$$m = \frac{2}{2} = 1$$

Slope Formula

- Pick your two points and find the coordinates
- Decide which one is Point 1 and which one is Point 2
- Use your coordinates in the slope formula
- DO NOT MIX UP THE ORDER OF YOUR POINTS OR BAD THINGS WILL HAPPEN
- Remember your units (if applicable)

8.0 - REVIEW

Slope Formula → Try with (5, 7) and (3, -4)

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

8.0 - REVIEW

Surface Area Formulas

- Rectangular Prism

- $SA = 2lw + 2lh + 2wh$

- Cylinder

- $SA = 2\pi r^2 + 2\pi dh$

- Cube

- $SA = 6s^2$

8.0 - REVIEW

- **Greatest Common Factor**
 - The highest number that divides evenly into all the values
 - Use a factor tree or write out the factors of each number and see what matches up

8.0 - REVIEW

Converting Measurements

- See Conversion Sheet
- Remember: If the unit is squared, “double the jumps”

8.0 - REVIEW

Exponent Laws

- When multiplying powers with the same base, add the exponents.
- When dividing powers with the same base, subtract the exponents.
- When a power is raised to a power, multiply the exponents.
- If an exponent is negative, it moves to the bottom of the fraction (and then it is positive and happy). $x^{-2} = \frac{1}{x^2}$

8.1 — COMPARING AND INTERPRETING RATES

Things to know:

- Always compare the same units (apples to apples)
- Unit rates should always be expressed as a number compared to ONE of the second unit (kilometers per one hour)
- Slope is the same as rate of change
- Slope represents average change

8.1 — COMPARING AND INTERPRETING RATES

Natasha can buy a 12 kg turkey from her Meats 'R' Us for \$42.89. Meat-opolis has turkeys advertised in its weekly flyer for \$1.49 per pound. There is approximately 2.2 lbs in 1 kg. Which butcher has the lower price?

12 kg \rightarrow lbs

$$\frac{2.2 \text{ lb}}{1 \text{ kg}} = \frac{x}{12 \text{ kg}}$$

$$x = 26.4 \text{ lbs.}$$

$$\left| \begin{array}{l} \frac{\$42.89}{26.4 \text{ lbs}} = \frac{\$1.49}{1 \text{ lb}} \\ \frac{\$1.62}{1 \text{ lb}} \quad \square \quad \frac{\$1.49}{1 \text{ lb}} \end{array} \right.$$

8.1 — COMPARING AND INTERPRETING RATES

1. Convert both to kg. Use cross multiplying.

\$1.49/lb

2.2 lbs per kg

8.1 — COMPARING AND INTERPRETING RATES

2. Find the Meats 'R' Us unit rate. (Divide)

\$42.89 for 12 kg

8.1 — COMPARING AND INTERPRETING RATES

3. Compare. Write a fun sentence.

8.1 — COMPARING AND INTERPRETING RATES

Pro Tip

- If I ask you a question in words, I expect to get words back.
- If I ask you a question with fractions, I expect to get fractions back.
- If I ask you a question with decimals, I expect to get decimals back.

8.1 — COMPARING AND INTERPRETING RATES

I want to buy a new car because my car is older than most of you and junky. But I love it.

Fuel efficiency is super important to me because I am cheap. Oh ya, and like saving the rainforest and stuff. (But mostly, I'm just cheap.)

8.1 — COMPARING AND INTERPRETING RATES

The gas tank of the black car I like has a capacity of 55 L. The owner's manual claims that it has a fuel efficiency of 7.6L/110km on the highway. I am skeptical, so I took it for a spin on the highway. I set my odometer to 0 km. I started with a full tank. Each time I stopped for gas, I recorded distance and gas.

8.1 — COMPARING AND INTERPRETING RATES

Fill Up	Total Distance (km)	Quantity of Gas (L)
1	645	48.0
2	1037	32.1

8.1 — COMPARING AND INTERPRETING RATES

When was my fuel efficiency the best, and was the owner's manual right?

8.1 – COMPARING AND INTERPRETING RATES

Car Manual
7.6L/110km

1. Calculate the efficiency per leg of the trip.

$$FE = \frac{\text{gas used}}{\text{distance driven}}$$

Fill Up	Total Distance (km)	Quantity of Gas (L)
1	645	48.0
2	1037	32.1

8.1 — COMPARING AND INTERPRETING RATES

2. Compare the rates.

First

$$\frac{48.0 \text{ L}}{645 \text{ km}}$$

$$= 0.0744 \text{ L/km}$$

x more fuel
efficient

Second

$$1037 - 645 = 392$$

$$\frac{32.1 \text{ L}}{392 \text{ km}}$$

$$= 0.0819 \text{ L/km}$$

8.1 – COMPARING AND INTERPRETING RATES

3. Put the manufacture's FE rating as a unit rate (or convert the other rates to be over 100 km), and then compare.

$$\begin{array}{r} \text{owner's manual} \\ 7.6 \text{ L} \\ \hline 110 \text{ km} \\ \hline = 0.0691 \text{ L/km} \end{array}$$

The owner's
manual measured
the efficiency lower.

8.2 — SOLVING PROBLEMS THAT INVOLVE RATES

Stuff You'll Know

- How to compare rates
- Analyzing and Solving Problems that Involve Rates

8.2 — SOLVING PROBLEMS THAT INVOLVE RATES

I want go shopping in Minot. My car can make it to Minot and back on a full tank (90 L). Gas here is \$1.06/L. South of the boarder, gas costs \$2.86 US/gal. As I said, I am cheap.

Would it be wise to fill up in the State while I can, since everything seems to be cheaper there?

8.2 — SOLVING PROBLEMS THAT INVOLVE RATES

1. Figure out how much a tank of gas costs in Canada.

\$1.06/L for 90 L

$$1.06 (90) = \$ 95.40$$

/ 90L

8.2 — SOLVING PROBLEMS THAT INVOLVE RATES

2. Convert the gallons and litres.

$$1 \text{ gal} = 3.97 \text{ L}$$

$$\frac{3.97 \text{ L}}{1 \text{ gal}} \quad \begin{array}{c} \xrightarrow{\quad} \\ \text{X} \end{array} \quad \frac{90 \text{ L}}{\text{X}}$$

$$\begin{array}{l} 3.97x = 90 \\ \hline 3.97 \\ x = 22.67 \text{ gal} \end{array}$$

$$\begin{array}{l} (\$ 2.86/\text{gal})(22.67) \\ = \$ 64.84 \\ \text{USD / gal} \end{array}$$

8.2 — SOLVING PROBLEMS THAT INVOLVE RATES

3. Figure out how much it costs to fill up in USA. \$1 USD = \$1.02 CAD. (multiply)

$$\frac{1.02 \text{ CAD}}{1.00 \text{ USD}} = \frac{X}{64.84 \text{ USD}}$$

$$X = \$66.14 \text{ CAD/tank}$$

90L

8.2 — SOLVING PROBLEMS THAT INVOLVE RATES

4. Compare. Write a fun sentence.

US
\$ 66.14

CAD
\$ 95.40

It is way cheaper to fill up in Murica.

8.2 — SOLVING PROBLEMS THAT INVOLVE RATES

In partners, describe three possible situations for each unit rate. We'll discuss as a class.

- **0.05 mg/kg** — weight on a different planet — trail mix ratio
— real meat in dog food
- **98.5 cents/L** — gas — water price — orange juice
- **7.2 MBps (Megabits per second)** — download speed
— processing speed
— upload speed

8.2 — SOLVING PROBLEMS THAT INVOLVE RATES

Herbert is a player. This year alone, he's gone out with 180 ladies. For Valentine's Day, he wants to order roses for each date. Some were better than others, so he estimates he'll need about 2.5 roses per lady. Roses come in bouquets of 24. How many bouquets should he buy?

8.2 — SOLVING PROBLEMS THAT INVOLVE RATES

Set up your formula with units. Solve. Write your fun sentence.

$$\frac{2.5 \text{ roses}}{1 \text{ lady}} = \frac{r}{180 \text{ ladies}} \qquad \frac{450}{24} = 18.75$$

$$r = 450 \text{ roses}$$

Herbert should buy 19 bouquets
and go on fewer dates.

8.2 — SOLVING PROBLEMS THAT INVOLVE RATES

Lepage and Gelinas are having a contest. Lepage calculated that he burns 256 calories in 20 minutes of running 10 km/h with no incline. Gelinas calculates that she burns 201 calories in 15 minutes at 8km/h with an incline of 6. After one hour of competition, who burned the most calories?

8.2 — SOLVING PROBLEMS THAT INVOLVE RATES

How much longer would Lepage have to run to catch up to Gelinas?

$$\begin{array}{l}
 \text{Lepage} \\
 \hline
 256c \\
 \hline
 20 \text{ min}
 \end{array}
 \quad
 256 \times 3 = 768 \text{ cal}
 \quad
 \begin{array}{l}
 \text{Gelinas} \\
 \hline
 201 \text{ cal} \\
 \hline
 15 \text{ min}
 \end{array}
 \quad
 201 \times 4 = 804 \text{ cal.}$$

$$\frac{256 \text{ cal}}{20 \text{ min}} = \frac{804 \text{ cal}}{x}
 \quad
 \left|
 \begin{array}{l}
 256x = 16080 \\
 \hline
 256 \\
 \hline
 x = 62.8125
 \end{array}
 \right.$$

QUIZ ANNOUNCEMENT

Quiz on 8.1 – 8.2 coming up! Be prepared.

If you need extra practice, check out p. 473 of the textbook. Ask me for a copy if you want it.

8.3 — SCALE DIAGRAMS

Things You'll Know

- 2D Scale Diagrams of stuff
- When to use a scale diagram
- How to interpret a scale diagram

8.3 — SCALE DIAGRAMS

Terms

- **Scale diagram**
 - A drawing in which the measurements are proportionally reduced or enlarged from actual measurements; a scale diagram is similar to the original.

8.3 — SCALE DIAGRAMS

- **Scale**

- The ratio of a measurement on a diagram to the corresponding distance measured on the shape or object represented by the diagram

- **Scale Factor**

- A number created from the ratio of any two corresponding measurements of two similar shapes or objects, written as a fraction, a decimal, or a percentage

$$k = \frac{\text{diagram}}{\text{original}}$$

8.3 — SCALE DIAGRAMS

Bob the Builder is building a house on a rectangular lot, as shown on the sketch. Draw a scale diagram of the lot and house using a scale of 1m : 500m.

diagram: original

8.3 — SCALE DIAGRAMS

$$R = \frac{d}{o}$$

$$\frac{1}{500} = \frac{x}{15}$$

$$500x = 15$$

$$x = 0.03\text{m}$$

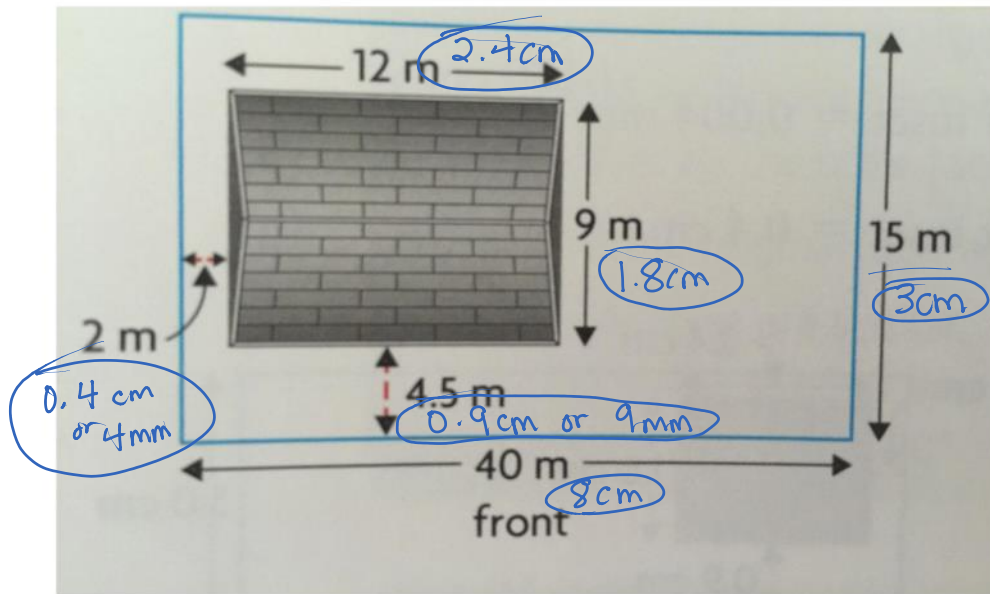
$$x = 3\text{ cm}$$

$$\frac{1}{500} = \frac{x}{9}$$

$$500x = 9$$

$$x = 0.018\text{m}$$

$$x = 1.8\text{ cm}$$



8.3 — SCALE DIAGRAMS

Calculate the dimensions of each side.

8.3 — SCALE DIAGRAMS

Draw it using a ruler.

8.3 — SCALE DIAGRAMS

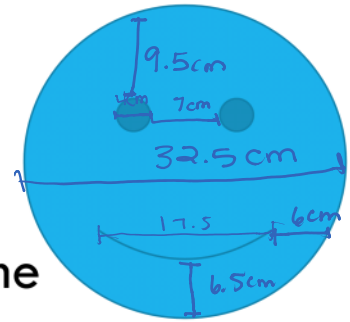
$$9.5 \times \frac{5}{3} = 15.83 \text{ cm} \quad \left| \quad 32.5 \times \frac{5}{3} = 54.17$$

$$7 \times \frac{5}{3} = 11.67 \text{ cm}$$

You need to enlarge this by a scale

Factor of $\frac{5}{3}$. $R = \frac{5}{3}$

1. Measure it.
2. Multiply your measurements by the scale factor.
3. Use your new measurements to draw the enlarged diagram.



8.3 — SCALE DIAGRAMS

The standard size of a stop sign measures at 30 inches across an octagon shape. There is a white border of 20 mm (or a little less than an inch) around the sign. While the octagon is the most recognized shape, used by most other countries around the world, some regions, such as Zimbabwe and Japan, opt to to use a circular or triangular stop sign.

Read more

: http://www.ehow.com/about_5565871_measurements-stop-sign.html

8.3 — SCALE DIAGRAMS

What is the scale factor of this stop sign?

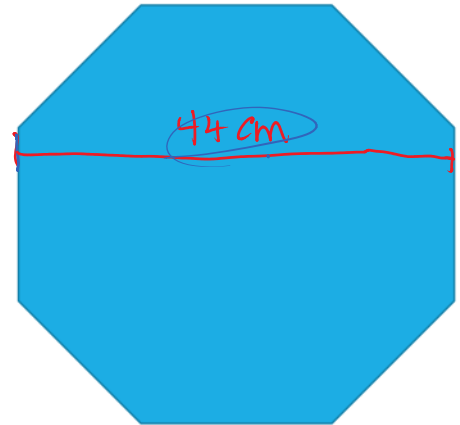
$$30 \text{ in} \rightarrow \text{cm}$$

$$30 \times 2.53 \text{ cm} = 75.9 \text{ cm}$$

$$k = \frac{d}{o}$$

$$k = \frac{44}{75.9}$$

$$k = 0.58$$



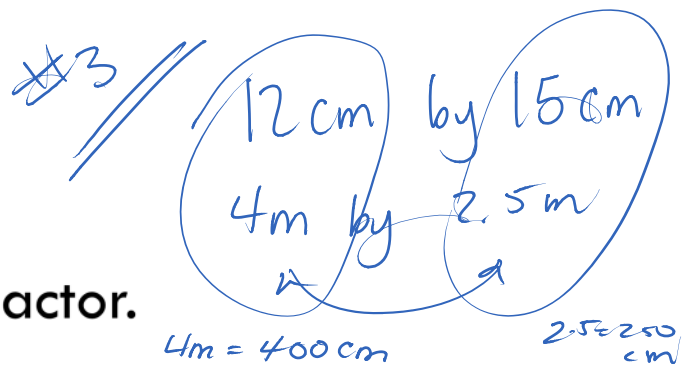
NEED TO KNOW

Let k represent the scale factor.

$$k = \frac{\text{Diagram Measurement}}{\text{Actual Measurement}}$$

$$15 \div 0.048 = 312.5 \text{ cm}$$

$$12 \div 0.0375 = 320 \text{ cm}$$



$$k = \frac{15}{400} = 0.0375$$

$$k = \frac{12}{250} = 0.048$$

$$k = 0.048 = \frac{48}{1000}$$

8.4 — SCALE FACTORS AND AREAS

You Will Know

- How to solve area problems that involve similar 2D Shapes

ProTips: k is LARGER than 1 \rightarrow enLARGEs it
 k is smaller than 1 \rightarrow shrinks it

8.4 — SCALE FACTORS AND AREAS

I want to fly a kite in this “glorious” weather. I want to make one because I’m cheap. The area of the scale diagram of the kite I want is 20 cm^2 . If the scale factor is 2:25, how much fabric will I actually need?

8.4 — SCALE FACTORS AND AREAS

*if the
Units are

To do this, you need to multiply the scale factor **SQUARED** by the area. It helps to write it as a fraction or decimal. (25/2 or 12.5) ^{Squared, Square the scale factor.}

-- remember this number is going to be larger than one since we are enlarging.

$$20 \text{ cm}^2 (12.5)^2 = 3125 \text{ cm}^2 \\ = 0.3125 \text{ m}^2$$

8.4 — SCALE FACTORS AND AREAS

What is the scale factor of the area of my laptop screen to my projected screen?

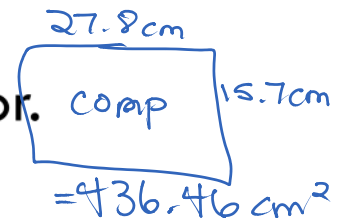
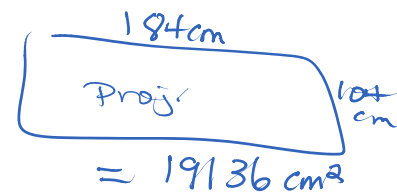
Remember:

$$k = \frac{\text{Diagram Measurement}}{\text{Actual Measurement}}$$

and area has a squared scale factor.

$$k^2 = \frac{19136 \text{ cm}^2}{436.46 \text{ cm}^2} = 43.8437 \rightarrow k = 6.6215$$

take sq. root



8.4 — SCALE FACTORS AND AREAS

1. Measure each and calculate the area.

8.4 — SCALE FACTORS AND AREAS

2. Divide the projected area by the laptop area. Set it equal to the scale factor SQUARED. Solve for k .

8.5 — SCALE MODELS AND SCALE DIAGRAMS

You'll Know This

- Use scale models and scale diagrams that involve 3D objects
- Know the math behind the aforementioned stuff

8.5 — SCALE MODELS AND SCALE DIAGRAMS

Helen is a chef. She has two frying pans, and OBVIOUSLY the most important question on her mind is, “Are they similar shapes?” What else would you ask about frying pans. Duh.

8.5 — SCALE MODELS AND SCALE DIAGRAMS



Pan	Height	Diameter	Handle Length
Big Pan	6 cm	30 cm	24 cm
Little Pan	4 cm	20 cm	16 cm

→ Calculate the linear scale factor (just plain old “k”) and check each dimension to make sure it matches up.

$$k = \frac{6}{4}$$

$$k = \frac{3}{2} \text{ or } \underline{1.5}$$

$$k = \frac{30}{20}$$

$$k = \frac{3}{2} \text{ or } \underline{1.5}$$

$$k = \frac{24}{16}$$

$$k = \frac{3}{2} \text{ or } \underline{1.5}$$

8.5 — SCALE MODELS AND SCALE DIAGRAMS

Okay. So, now that we know the scale factor is $\frac{3}{2}$. If the lid handle of the big pan is 9 cm long, how long is the lid handle of the little pan?

$$\begin{array}{l} \frac{3}{2} \rightarrow \frac{9 \text{ cm}}{x} \\ \frac{3x}{3} = \frac{18}{3} \\ x = 6 \text{ cm} \end{array}$$

8.5 — SCALE MODELS AND SCALE DIAGRAM

I bought the Karlstad from Ikea.

The instructions are at a scale of $1:15$. The dimensions of the diagram are $1.6'' \times 1.6'' \times 6''$. What are the actual dimensions, (and will I actually use all the screws to build it)?

$$\begin{aligned} 1.6 \times 15 &= 24'' \\ 1.6 \times 15 &= 24'' \\ 6 \times 15 &= 90'' \end{aligned}$$



8.5 — SCALE MODELS AND SCALE DIAGRAMS

1. Multiply (because we are getting bigger) by the scale factor. Remember Diagram:Actual. Do this for each measurement.

8.5 — SCALE MODELS AND SCALE DIAGRAMS

My friend Kaylee is working on a project downtown to replace the pipes that contain the wiring for downtown. To do this, she needs to draw diagrams for the city. For her project, she determined that each pipe has to have an inner diameter of 1.5 m, length of 2.5 m, and wall thickness of 0.18 m. How can she create a scale drawing of these pipes?

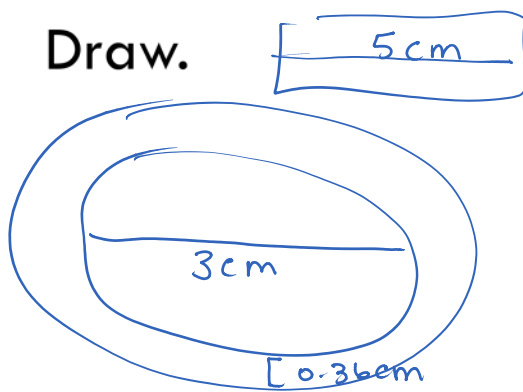
$2 \text{ cm} : 1 \text{ m}$
 $2 \text{ cm} : 100 \text{ cm}$
 $1 \text{ cm} : 50 \text{ cm}$

8.5 — SCALE MODELS AND SCALE DIAGRAMS

Choose a “view.” (Or views)

Find an appropriate scale factor.

Draw.



$$\frac{50}{1} = \frac{250}{x}$$

$$50x = 250$$

$$x = 5$$

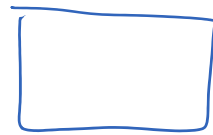
$$\frac{50}{1} = \frac{18}{x}$$

$$50x = 18$$

$$x = 0.36\text{cm}$$



Front



Side

$$\frac{50\text{cm}}{1\text{cm}} = \frac{150}{x}$$

$$50x = 150$$

$$x = 3\text{cm}$$

8.6 — SCALE DRAWING OF 3D OBJECTS

You'll Know:

- How to solve problems with scale factor, surface area, and volume.

[HTTP://WWW.101QS.COM/3043](http://www.101qs.com/3043)

8.6 — SCALE DRAWING OF 3D OBJECTS

Lindt wants to build a replica of the Pyramid of Giza in Vegas made completely out of chocolate. The space they have is restricted to 6 m by 6m and 3.5 m high. Keep in mind that they need a 1 m clearance path all the way around.

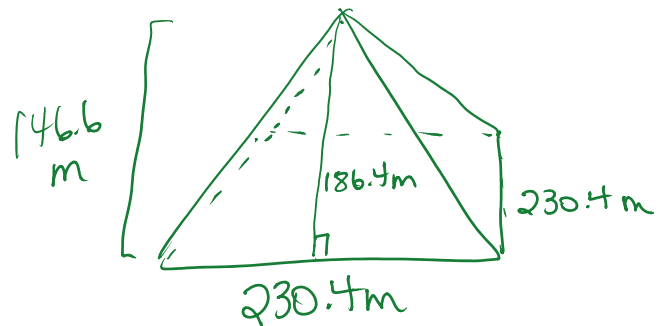
8.6 — SCALE DRAWING OF 3D OBJECTS

The actual size of the pyramid is 146.6 m high, 186.4 m slant, and 230.4 m base length.

What scale factor should Lindt use?

8.6 — SCALE DRAWING OF 3D OBJECTS

1. Draw a diagram.



8.6 — SCALE DRAWING OF 3D OBJECTS

2. Compare the values you have in common to find scale factor.

In this case, compare the base side length.

$k = \frac{\text{original}}{\text{replica}}$

$$k = \frac{230.4}{5} = \underline{46.08}$$

$$k = \frac{146.6}{3.5} = \underline{41.89}$$

$\begin{array}{l} 6 \times 6 \times 3.5 \\ \downarrow \\ 5 \times 5 \times 3.5 \end{array}$

8.6 — SCALE DRAWING OF 3D OBJECTS

3. Use your scale factor on the other dimensions.

$$5 \times 41.89 = 209.45 \text{ m} \rightarrow \text{base}$$

$$3.5 \times 41.89 = 146.62 \text{ m} \rightarrow \text{height.}$$

$$186.4 \div 41.89 = \boxed{4.45 \text{ m}}$$

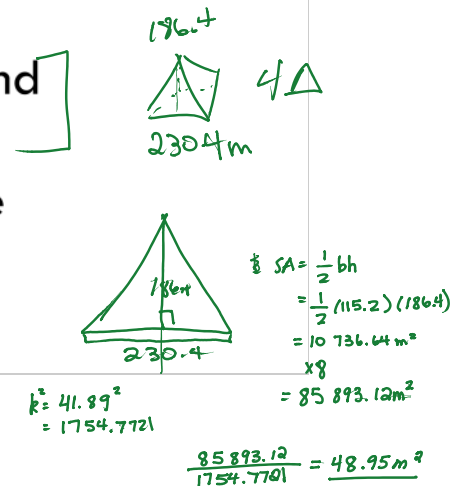
8.6 — SCALE DRAWING OF 3D OBJECTS

If the sculpture was only chocolate covered, how much chocolate would they need?

Two routes:

Calculate the surface area of the original and divide by k squared.

OR Use your new dimensions to calculate the surface area.



Handwritten calculations and diagrams:

Top diagram: Square pyramid with base side 186.4 and slant height 230.4. A bracket indicates the calculation of the surface area of the original.

Bottom diagram: Triangle representing the lateral face with base 230.4 and height 186.4.

Calculations:

$$k^2 = 41.89^2 = 1754.7721$$

$$SA = \frac{1}{2}bh = \frac{1}{2}(115.2)(186.4) = 10736.64 \text{ m}^2$$

$$\times 8 = 85893.12 \text{ m}^2$$

$$\frac{85893.12}{1754.7721} = 48.95 \text{ m}^2$$

8.6 — SCALE DRAWING OF 3D OBJECTS

Two water balloons are inflated. The smaller balloon has a capacity of 140.0 cm^3 . The larger balloon has a capacity of 472.5 cm^3 . Both are filled with water at the same rate. How many times longer will it take to fill the larger one?

8.6 — SCALE DRAWING OF 3D OBJECTS

Divide the volume of the larger balloon by the volume of the smaller balloon. This will give your scale factor for volume (k^3).

$$k^3 = \frac{472.5}{140}$$

$$k^3 = 3.375$$

8.6 — SCALE DRAWINGS OF 3D OBJECTS

How many times larger is the radius of each?
(Take the cube root of the scale factor for the linear scale factor.)

$$\sqrt[3]{k^3} = \sqrt[3]{3.375}$$
$$k = 1.5$$

TIPS

Remember that when you are trying to decide if you find the linear scale factor, the square scale factor, or the cubed scale factor, look at the units you are using.

You can also remember that linear measurements are just k. Area is squared. Volume is cubed.

$$\begin{aligned} \text{cm}^1 &\rightarrow k^1 \\ \text{cm}^2 &\rightarrow k^2 \\ \text{cm}^3 &\rightarrow k^3 \end{aligned}$$

ANNOUNCEMENTS

Quiz on Feb. ~~11~~ 10

8.3-8.6

~~Start Project on Feb. 12~~

Review - Feb 10/11

~~Project due on Feb. 23 (go get a handout
when you finish your quiz)~~

Test - Feb. 12.