Ch 6- Systems of Linear Inequalities

6.1- Graphing Linear Inequalities in Two Variables

Let's start with a little review. What can you tell me about:

Natural Numbers: 1, 2, 3, 4, 5, ...

Whole Numbers: 0, 1, 2, 3, 4, 5, ...

Integers: ... -3, -2, -1, 0, 1, 2, 3, ...

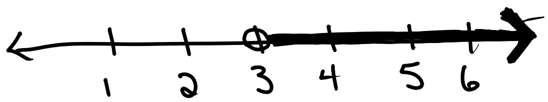
A mathematical inequality must contain one of the following symbols:

$<$ $>$ $\leq$ $\geq$ $\neq$

The following are examples of linear inequalities in a single variable:

$x > 3$

The solution to a single variable inequality can be shown on a number line:



6.1: Types of Line/Shading for Inequality Graph

- Solid line:
- Stippled line:
- Dashed line:
- Full shading:
- Stippled:

*Check for which side to stipple or shade by using a test point [(0, 0) if it isn't on the line]

Example 1. Graph the solution set for $-3x + 4y \leq 12$

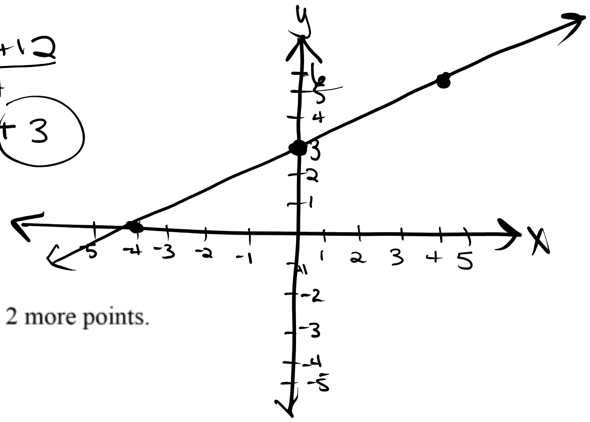
1) Solve for y.

$$\overset{+3x}{-3x} + 4y \leq 12 \; {}^{+3x}$$

$$\frac{4y}{4} \leq \frac{3x+12}{4}$$

$$y \leq \frac{3}{4}x + 3$$

2) Plot the y-intercept.



3) Use the slope to get 2 more points.

Example 2. Graph the solution set for each linear inequality on a Cartesian plane.

a) $\{ (x, y) \mid x - 2 > 0, x \in R, y \in R\}$

b) $\{ (x, y) \mid -3y + 6 \geq -6, , x \in I, y \in I\}$

Example 3. Oliver and Connor are competing in a spelling quiz. Connor gets a point for every word he spells correctly. Oliver is younger than Connor, so he gets 3 points for every word he spells correctly, plus one bonus point. What combinations of correctly spelled words will allow Connor to win? Choose two combinations that make sense and explain your choices.

SETS

- A **set** is a collection of objects enclosed in braces, { }.
- The objects are called **elements**; $\in$ is the symbol for "is an element of".
- The **empty set** has no elements; Ø is the symbol for the empty set.
- The **union** of two sets is the set containing elements that come from either set; $\cup$ is the symbol for union.
- The **intersection** of two sets is the set with elements that must be in both sets; $\cap$ is the symbol for intersection

Example: Given sets A = {h, i, t} and B = {g, i, s, t} and C = {j, o, y}

a) elements h $\in$ A, j $\in$ C, s $\in$ B
b) A $\cup$ B = {g, h, i, s, t} A $\cap$ B = {i, t}
c) A $\cup$ C = {h, i, j, o, t, y} A $\cap$ C = Ø { }

NUMBER SETS

Numbers are classified into sets according to common characteristics.

- Natural Numbers, $\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$
- Whole Numbers, $W = \{0, 1, 2, 3, 4, 5, \ldots\}$
- Integers, $I = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
- Rational Numbers, $\mathbb{Q} = \{x \mid x = a/b,\ a \in I,\ b \in I,\ b \neq 0\}$
- Irrational Numbers = {all real numbers that are not rational}
- Real Numbers $\mathbb{R}$ = {all numbers that are on the number line}

<u>Mathematical Statements:</u>

- An **equation** states that two expressions have the same value
- An **inequality** compares the values of two expressions
- A **compound statement** combines two statements using "and" or "or"
- Solving a mathematical statement means to find all the values that "satisfy" the statement, to find all the values for which the statement is true.
- The **solution set** can be used to describe the solution of a statement
- A **graph**, either on a **number line** or on a **coordinate plane** can be used to illustrate the solution set.

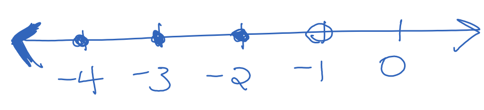
Example: Write the solution set and then graph the solution on a number line

a) $3 - 2x > 5$, where $x \in I$
   - Solve like an equation
   - Multiply/divide by a negative, reverse inequality

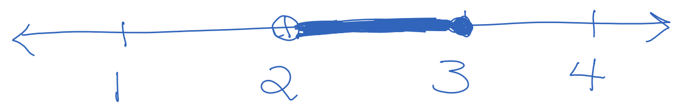
$$\underset{-3}{3} - 2x > \underset{-3}{5}$$

$$\frac{-2x}{-2} > \frac{2}{-2}$$

$$x < -1$$



b) $x > 2$ and $x \leq 3$, where $x \in \mathbb{R}$

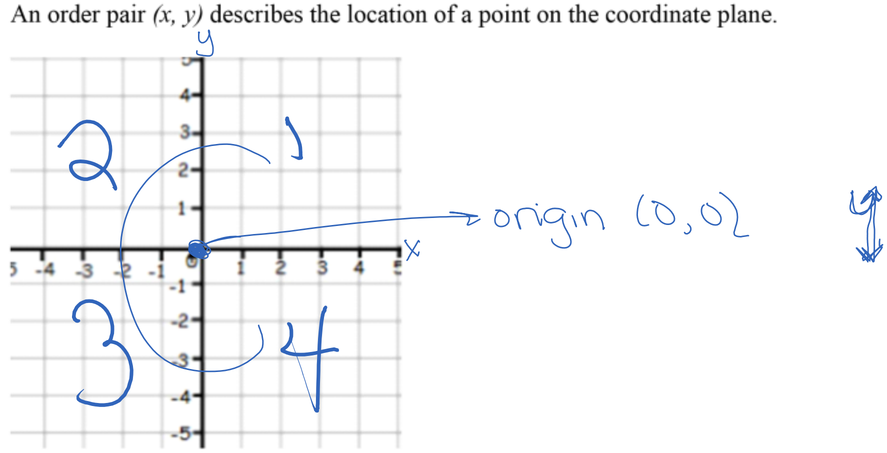


The Coordinate Plane:

On the coordinate plane shown, label:

- The origin
- The x and y axis
- Quadrant I-IV

An order pair *(x, y)* describes the location of a point on the coordinate plane.



Relations

- A relation describes the relationship between two quantities, x and y are usually used to represent the two quantities, but other variables can be used.

Example. The price of coffee is \$3. The relationship between the cost and the number of cups purchased can be described:

- in words:

  every cup cost \$3.

- with an equation:

  Let P be the price of coffee

  Let c be the number of cups.

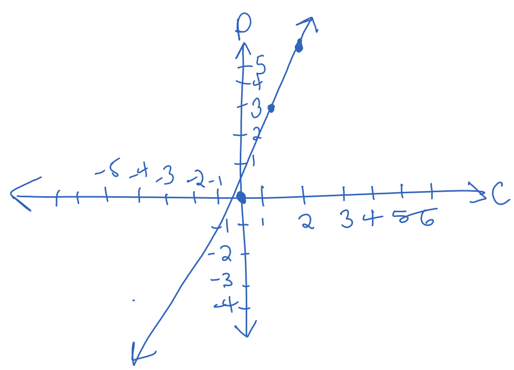
  $$P = 3c$$

- as a set of ordered pairs: (c, P)

  $(0,0)$ $(2,6)$ $(4,12)$ ...

  $(1,3)$ $(3,9)$

- with a graph:



Linear Relations

Equations of all linear relations can be written in the form $Ax + By = C$.

The ***x*- intercept** is the point on the linear graph that is also on the *x*-axis.

- The x-intercept will be point (a, 0) or just the number a.

The y- intercept is the point on the linear graph that is also on the *y*-axis.

- The *y*-intercept will be point (0, b) or just the number b.

Example: Calculate the *x*-intercept of $2x - 3y = 6$ without graphing.

- The x-intercept is when y = 0.

$2x - 3(0) = 6$
$2x = 6$
$x = 3$

(3, 0)

$\cancel{2(0)} - 3y = 6$
$-3y = 6$
$y = -2$

(0, −2)
y-int

The **slope-intercept form** of a linear relation's equation is $\boldsymbol{y = mx + b}$

- The slope; $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

- b is the y-intercept of the function.

Example: Determine the slope and y-intercept of the straight line graph described by $2x - 3y = 6$ and then graph the relation.

1. Write the equation in slope-intercept form, $y = mx + b$

$2x - 3y = 6$
$-2x \quad\quad -2x$

$\dfrac{-3y}{-3} = \dfrac{-2x + 6}{-3}$

$y = \dfrac{-2}{-3}x + \dfrac{6}{-3}$

$y = \dfrac{2}{3}x - 2$

$m = \dfrac{2}{3}$

y-int = (0, −2)

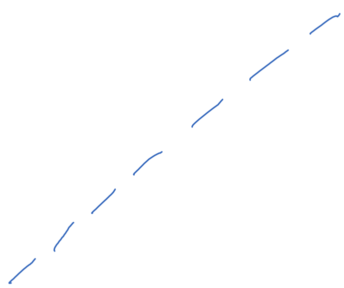
* Watch signs! 6o6

<u>A linear inequality in two variables</u> is a mathematical statement like a linear equation in two variables with the equal symbol replaced with an inequality symbol, $>, <, \leq, \geq$.
The solution is the set of all ordered pairs that satisfy the statement.

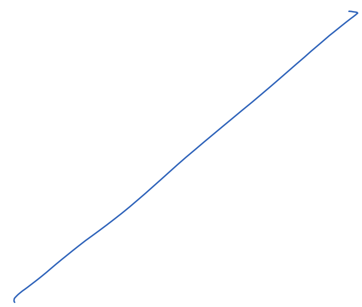
Consider the graph of the equation $2x - 3y = 6$

- All the points on the line have coordinates that satisfy the equation
- For all the other points, $2x - 3y$ will <u>not</u> equal 6
    - (-2, 2) is on one side of the line. For this point, $2x - 3y$ is less than 6. All the points on this side of the line satisfy the inequality $2x - 3y < 6$.
    - (3, -3) on one side of the line. For this point, $2x - 3y$ is greater than 6. All the points on this side of the line satisfy the inequality $2x - 3y > 6$.

The graph of an inequality in two variables must represent all (and only) the points with coordinates that satisfy the statement.



dashed lines
points on it do NOT satisfy the statement



solid line
points on it also satisfy the statement

<u>Graphing a Linear Inequality in Two Variables</u>

- Draw the **boundary line**:
    - Dashed or solid? Dashed for > or < ; Solid for ≥, or ≤ "equal to" Solid line under the sign
    - Where? Graph of the corresponding equation
- Determine the **half-plane** containing the solution:
    - select a test point on one side of the boundary- if it satisfies the inequality the solution is on this side; if not, the solution is on the other side use origin when possible
    - if domain and range are real numbers, shade that half-plane; if the domain and range are integers, **"stipple"** the half-plane with discrete points.

Examples: